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Critical relaxation of the eight-vertex model

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Abstract. The dynamical critical exponent z is obtained through a Monte Carlo Renormalisation Group calculation for the eight-vertex model with non-conserved dynamics. The value of z at several points along the critical line is evaluated. No appreciable variation in z relative to the Ising value is observed within the accuracy of our work.

1. Introduction

In two dimensions, various calculations yield the dynamical critical exponent z for simple Ising-like systems with conserved and non-conserved dynamics. A large group of systems are expected to be in the same dynamical universality class. It is an interesting question whether this class also includes models which are not in the same static universality class (Suzuki 1974, Forgacs *et al* 1980). The question is also relevant to certain two-dimensional systems which can be studied experimentally and are expected to have non-universal static critical exponents (see, for example, Roelofs and Estrup 1983). Improved experimental techniques are expected to provide a measurement of z in these systems. In this work, we have studied the critical dynamics of the eight-vertex model (with non-conserved order parameter dynamics) which is known to have non-universal static critical exponents (Baxter 1973), with the Monte Carlo Renormalisation Group (MCRG) method.

MCRG method was introduced by Ma (1976) as a technique to evaluate static and dynamic critical exponents. The method was improved by Swendsen (1979) and later by Wilson (1982, unpublished) and Tobochnick *et al* (1981). Recently, Jan and Stauffer (1982) (hereafter referred to as JS) have introduced the dynamical scaling concept to the MCRG, resulting in a simpler procedure.

In this work, we have used a modified form of the JS method to estimate the exponent z at several critical points (one of which corresponds to that of the Ising model as a special case) of the eight-vertex model. We could not observe an appreciable change in z within the accuracy of our computation.

In § 2 the details of the method are given. The model is specified in § 3. Section 4 describes the computational work and the results are presented in § 5.

2. The method

Several methods are now available for the computation of the dynamical critical exponent z in classical spin systems. Among these tools, ones that utilise the Monte Carlo simulation are easily applicable to a wide range of models. Simulating the system

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near the critical temperature T_c and observing how the response time τ of the system changes as a function of temperature T is the most direct method of estimating a value for z through the asymptotic relation

$$\tau \simeq |T - T_c|^{-\nu z}, \tag{1}$$

where ν is the correlation length exponent. However, the amount of computational effort required to obtain z (for a given accuracy) is reduced considerably if renormalisation group (Ma 1976, Swendsen 1979, Wilson 1982 unpublished, Tobochnick *et al* 1981 and Jan and Stauffer 1982), or finite size scaling (Yalabik and Gunton 1979) methods are combined with the simulation.

The MCRG method introduced by Ma, improved by Swendsen and later by Wilson has been applied to a variety of systems (Tobochnick *et al* 1981, Jan and Stauffer 1982, Yalabik and Gunton 1982, Katz *et al* 1982, Shenker and Tobochnick 1980). All the above versions of the method use the Monte Carlo simulation to evaluate various time dependent correlations of systems near equilibrium, which are in turn used to estimate z.

Recently, JS have introduced a MCRG method which estimates z from the way the system relaxes *to* equilibrium, using dynamical scaling (Jan and Stauffer 1982, Jan *et al* 1983). In particular, they study the way the magnetisation m of the system and its renormalised forms relax from their saturation values. The dynamical scaling hypothesis asserts that at criticality the aforementioned magnetisations are related to one another by

$$m(t) \sim m'(tb^z), \tag{2}$$

where m' is the renormalised magnetisation, t the time and b the renormalisation scale change. By studying the relatively early time behaviour of m and m', JS produce an accurate estimate of z for the two-dimensional Ising model with a non-conserved parameter (Glauber model). In this work, we apply this method to a special case of the eight-vertex model (again with a non-conserved order parameter), although we use a somewhat more involved analysis of the Monte Carlo data than that implied by equation (2).

3. The model

The model we have studied can be defined through a Hamiltonian H such that

$$-H/kT = K \sum_{\langle ij \rangle} S_i S_j + Q \sum_{\langle ijkl \rangle} S_i S_j S_k S_l, \tag{3}$$

where k is the Boltzman constant and $\{S_i\}$ denote spin variables which can take values ± 1 on a square lattice. K and Q correspond to two-spin and four-spin coupling constants respectively and the first sum is over all next nearest neighbour pairs, while the second is over all spin quads on a unit square. For $Q = 0$, the model decomposes into two independent Ising models with nearest neighbour coupling K. Baxter (1973) has shown that the model exhibits criticality along a line on the K–Q plane defined by

$$\sinh 2K = \exp(-2Q) \tag{4}$$

with non-universal static critical exponents along the line. In particular

$$\nu = \pi/2\bar{\mu} \qquad \beta = \pi/16\bar{\mu} \tag{5}$$

where $\bar{\mu}$ is given by

$$\cos \bar{\mu} = \tanh 2Q \tag{6}$$

and β is the order parameter exponent.

We associate with this model a non-conserved order parameter dynamics in which spins flip with a transition probability which guarantees an equilibrium distribution that corresponds to the Hamiltonian H. We simulate this dynamics by the standard Monte Carlo procedure (Binder 1976) in which a site is chosen at random, and the spin at that site is flipped if the new configuration corresponds to a lower energy. Otherwise the spin is flipped with probability $\exp(-\Delta E/kT)$, where ΔE is the increase in the energy of the system upon flipping the spin. Time is measured in Monte Carlo steps (MCS) which corresponds to the average number of spin flip attempts performed per spin.

4. Computations

Computations were carried out on a 162 by 162 lattice (using multi-spin coding). The system was relaxed from the fully magnetised state and the magnetisations of the original as well as the renormalised lattices were computed after every MCS. Renormalisations of the lattice were performed by applying the majority rule and the blocking scheme shown in figure 1 (resulting in a scale change of $b = 3$). Although the lattice was renormalised several times, it was found that only the original and once-renormalised lattices provide usable magnetisation values (JS reported no significant improvement in accuracy by utilising higher order renormalisations). The computations were carried out at four critical points corresponding to Q values of -1, -0.3, 0 and 0.3. Eleven independent runs were performed at each Q value and the average of these runs were used in further analysis. The difference in our method from that of the original method of JS is in the way we analyse the magnetisation data. Instead of equation (2), which is expected to be true asymptotically for sufficiently large times, we assume the form

$$m(t-t_0) = m'[b^z(t-t_0')], \tag{7}$$

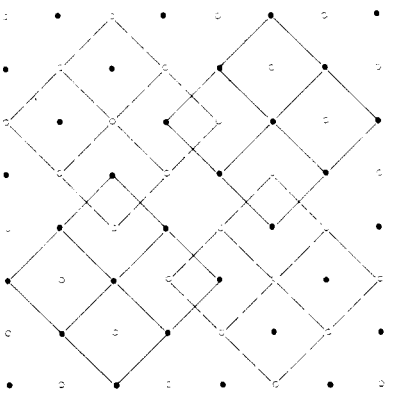

Figure 1. The blocking scheme used in the computations. Full and open circles correspond to the two sublattices of the original model while the 3×3 blocks constructed with continuous and broken lines correspond to the sublattices of the renormalised version.

where t_0 and t_0' are two constants representing a possible shift in the time scales of corresponding magnetisation functions. We believe that for the relatively small times for which we are analysing the data, equation (7) will result in a more reliable estimate for z. Furthermore, since both the original and the renormalised magnetisations are expected asymptotically to behave as (Suzuki 1977)

$$m \sim t^{-\beta/\nu z}, \tag{8}$$

equation (7) implies

$$m(t-t_0) = b^{-\beta/\nu} m'(t-t_0'). \tag{9}$$

The procedure for estimating z is then as follows.

(i) The magnetisation curves m and m' were shifted in time with respect to one another until their ratio was as close to $b^{-\beta/\nu}$ as possible over a specified interval, in conformity with equation (9) (note that $\beta/\nu = 1/8$ for all points on the critical line). This resulted in an optimal determination of $\Delta t = t_0' - t_0$. (Although in principle one could also estimate β/ν from the matching procedure, the errors introduced do not make it practicable.)

(ii) The magnetisation values were then fitted to the form of equation (9), using the remaining two adjustable parameters (one of which is z) over the same interval as in the first part of the procedure. Choice of the interval to be used in obtaining an estimate for z is somewhat ambiguous. We observe the same behaviour in the magnetisation functions as was reported by JS, namely, a fast decay in early times and

Table 1. Estimates of z obtained at various values of Q using different intervals. σ denotes a goodness-of-fit parameter. The intervals marked with an asterisk correspond to the subjective choices.

$Q=-1$								
Interval (MCS)	60–110	50–100*	60–120	50–110	50–120			
z	2.022	2.235	1.89	2.13	2.005			
σ	0.084	0.085	0.086	0.1	0.112			
$Q=-0.3$								
Interval (MCS)	60–110*	60–100	70–110	50–110	50–120	70–120	50–100	
z	2.24	2.206	2.34	2.298	2.37	2.51	2.298	
σ	0.068	0.071	0.071	0.08	0.083	0.085	0.088	
$Q=0.0$								
Interval (MCS)	85–135*	85–200	100–200	85–160	75–200	85–220	100–220	75–135
z	2.28	2.37	2.41	2.46	2.4	2.5	2.58	2.42
σ	0.055	0.058	0.06	0.066	0.068	0.076	0.078	0.08
$Q=0.3$								
Interval (MCS)	120–200*	100–200	85–200	100–220	120–220	130–220	85–220	
z	2.19	2.26	2.32	2.40	2.41	2.46	2.44	
σ	0.04	0.05	0.06	0.06	0.06	0.07	0.20	

oscillations at late times, sandwiching a 'usable' region in between. The determination of the boundaries of these regions is necessarily subjective. In order to test our choice of the optimal time interval, we have made a systematic investigation of various intervals as other candidates for analysis. Table 1 lists these intervals, corresponding z values, and a goodness-of-fit parameter associated with that particular value of z. Some intervals do not yield a best-fit value and are not included in the table. Our (subjective) choice of the optimal interval never ranks worse than second-best in terms of the goodness of fit. Hence we feel justified in choosing the corresponding z values which are displayed in figure 2.

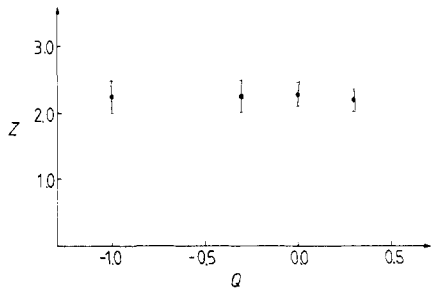

Figure 2. Estimates of z against the four Q values for which computations were performed. The length of an error bar corresponds to one standard deviation obtained from the eleven independent runs made at that particular Q value.

5. Results and conclusion

Within the accuracy of our work, we could not detect a variation in the dynamical critical exponent z over the range of Q values we have studied, at least one as dramatic as in the static exponents that do change. An invariance of the exponent z would be consistent with the weak universality hypothesis of Suzuki (1974, see also Forgacs *et al* 1980). A more accurate evaluation of z over a larger range of Q values is necessary to investigate the possibility of a weak variation of the exponent. More work is necessary on models similar to the one discussed in this study before any general remarks can be made on the extent of the dynamical universality class.

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